# A SIMPLIFICATION OF THE PRACTICE OF THE METHOD OF CONSTANT STIMULI*

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The history of the development of the method of constant stimuli has been one of exceeding interest. Starting with Fechner's method of right and wrong cases, it has now become standardized in a form that would hardly be recognised by one not acquainted with that development. The effort throughout has been toward greater exactitude, and the development has been along theoretical mathematical lines,—each advance, however, being verified by psychological experimentation. We now are able to handle the three categories of judgments, which introspection has shown to be the psychological facts.[1] Urban's notion of the psychometric functions[2] has done much to clear up our ideas about all of the psychophysical methods, but especially those of the method of constant stimuli.

Besides striving for greater exactitude and a better theoretical understanding of these methods, there has been a continuous effort to reduce and systematize the calculations required for the determination of the thresholds, with which all of the psychophysical methods are primarily concerned. In the very best and most systematized form of procedure of this method, the time required for the mere calculation of each threshold is from one and a quarter to one and half hours.[3] Such a period is too long to allow the method to be used for anything but a scientific study; it absolutely precludes its use for either anthropometric or diagnostic purposes. In fact few investigators have been willing to undertake the toil of such

* The experimental work that forms the basis of this paper was performed at the University of Pennsylvania in 1912. I am indebted to Professors F. M. Urban and J. W. Baird for many helpful suggestions offered in the preparation of the paper.

[1] Cf. E. B. Titchener, *Experimental Psychology.* II. Pt. 2, 1905. 286ff.

[2] F. M. Urban, *The Application of Statistical Methods to the Problems of Psychophysics.* Philadelphia, 1908. 106ff.

[3] Cf. S. W. Fernberger, On the relation of the methods of just perceptible differences and constant stimuli. *Psychol. Rev. Monographs,* XIV, No. 4. (Whole No. 61), 1913, 29-38.

calculations in spite of the great exactitude of the method and notwithstanding certain other obvious experimental advantages of this over the other psychophysical methods. Of these, the most important is that, with this method alone, the subject is kept in ignorance of the objective relations of the stimuli. The data required for the determination of the sensitivity of the subject consist in a number of empirical observations of the relative frequencies of the various judgments on several comparison stimuli equidistant from one another. From these data it is possible, by an application of the method of least squares, to calculate two constants—*h* and *c*—which determine the form and position of the curve representing the psychometric function for the different judgments. Once these values have been obtained it is possible to calculate the value of the threshold, which is defined as that point in the curve of the psychometric functions for the less or greater judgments, where the curve assumes a value one-half, or a probability of 0.5 that this judgment will be given. It is also possible to calculate, by means of these constants, the probability of a heavier or lighter judgment for any intensity of the comparison stimulus. The probability of obtaining an equality judgment is found by subtracting from unity the sum of the probabilities of the heavier and lighter judgments, as these are probabilities of mutually exclusive events. Hence the calculation of the constants *h* and *c* for the greater and less judgments determines the course and position of the curves of the psychometric functions for all three categories of judgment.

Such a calculation, however, requires the calculation of the quantities [P], [xP], [xxP], [$\gamma$P] and [x$\gamma$P],—values required for setting up the normal equations; and furthermore, two values [sP] and [xsP] which are needed to check up the other five values.[4] This process of setting up and solving the normal equations is very laborious. Even with the help of an adding machine it requires approximately one and a quarter hours; and the use of large multiplication tables and of logarithms is still more cumbersome.

One cause of this great expenditure of time is the size of the numbers that enter into the calculations. The x in every case represents the intensity of the comparison stimulus, for example, in the case of our experiments with lifted weights, 84, 88, 92 . . . 108 grams. Hence one is here obliged to handle two- and three-place numbers in a long and tedious,

[4] For a detailed description of this form of calculation *cf.* Fernberger, *ibid.*, p. 29-38.

even if not difficult, series of multiplications. Wirth[5] reduced this calculation considerably by reducing the size of the values of the term x. He reasoned that inasmuch as the values of x were always chosen equidistant, we may substitute the numbers 1, 2, 3 . . . n for these, without affecting the final values of the threshold. Furthermore, as one uses this series of digits in regular order, let us consider the central intensity of the comparison stimuli to be zero; and then for the stimuli smaller, substitute in order –1, –2, –3 . . . –n; and for the larger stimuli, 1, 2, 3 . . . n. In this manner, the values of the term x are greatly reduced. Urban[6] argues that, in this case, the values of x always remain the same, so that it is a simple matter to calculate all of the values for these five products for all possible frequencies of the judgment. This he does; and the tables which he obtains cover only the space of two pages and are therefore very handy and easily applied. In using these tables, one only has to look up the values of the products for the various observed relative frequencies and add each group, thus obtaining the quantities necessary for setting up the normal equations. Also, as the values in the tables are absolutely exact, it is no longer necessary to prove the correctness of the multiplications; and one only has to be sure that they have been transcribed accurately. With the aid of these tables, the calculation for a threshold takes from 15 to 20 minutes or even less.

It would seem then, that the work of the actual calculations has been reduced as much as possible. He must solve the normal equations, and any reduction must be in obtaining the quantities necessary for the setting up of these equations. It is impossible to see how this may be further reduced, beyond the tables of Urban's, mentioned above, as these require the mere transcribing of the appropriate values. Any further advance, therefore, must be in the line of reducing the amount of work required for making the observations. This may be done either by an improvement in the technique of experimentation or by reducing the number of comparison stimuli used. The question how many comparison stimuli should be used has not been satisfactorily settled. Of course, it would be desirable to use as many as possible; but the time as well as the energy of the investigators have their limitations. It seems that the question was decided chiefly by convenience. The usual number for experimental studies has been seven

[5] W. Wirth, *Psychophysik,* Leipzig, 1912. p. 213.

[6] F. M. Urban, Hilfstabellen für die Konstanzmethode. *Arch. f. d. ges. Psychol.,* XXIV, 1912, 236ff.

pairs.[7] Theoretically it should be possible to use two appropriately chosen comparison stimuli, because the empirical determination of two points furnish two equations for the calculation of $h$ and $c$, and should determine the course and position of the entire curve of the psychometric functions. For obvious practical reasons such an experimental procedure would be unwise; if for no other reason than that it would be impossible to keep the subject in ignorance of the objective relation in the stimuli presented.

In a former study, mentioned above, six pairs of comparison stimuli were employed, the seventh being an adjustable stimulus for the accumulation of data by the method of just perceptible differences.[8] A standard weight of 100 grams was the first member of each pair of stimuli. The second members, or the comparison stimuli, weighed 84, 88, 92, 96, 104 and 108 grams. The stimulus for the method of just perceptible differences replaced the usual 100 gram weight of this series. With these stimuli a series of 1,400 judgments were taken upon each of the pairs, two subjects being employed. For convenience, the results for each subject were divided into 14 groups of 100 each in the order in which they were taken.

An examination of the observed relative frequencies[9] shows that for both subjects, particularly after some little practice, the judgment 'lighter' is given upon the 84 gram weight, and the judgment 'heavier' upon the 108 gram stimulus with a very high relative frequency. Hence it seemed reasonable that these extreme values of the comparison stimuli might be disregarded without materially affecting the course of the curves of the psychometric functions. In this case the constants $h$ and $c$ which determine this course and which are also the values required in the calculation of the thresholds would not be materially affected.

As a result of this, we recalculated the data of our former study disregarding the two extreme values of the comparison stimuli, using only the four central terms,—namely 88, 92, 96 and 104 grams. This calculation was very simple, since the

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[7] L. J. Martin and G. E. Müller, *Zur Analyse der Unterschiedsempfindlichkeit,* 1899, 6.

F. M. Urban, *The Application of Statistical Methods to the Problems of Psychophysics,* 1908, 5ff.

S. W. Fernberger, On the relation of the methods of just perceptible differences and constant stimuli. *Psychol. Rev. Monographs,* XIV, No. 4, 6ff.

[8] Fernberger, *ibid,* pp. 6-15.

[9] Fernberger, *ibid,* pp. 20-21.

five products for each intensity of the comparison stimulus had been previously determined and their correctness proven. So all that was necessary was to obtain the sum of P, xP, xxP, $\gamma$P, and x$\gamma$P for the four central values, substitute these in the normal equations and solve for *h* and *c*. The quantities [P], [xP], [xxP], [$\gamma$P] and [x$\gamma$P] assumed very different values from the former calculation; but as these are not significant they may well be omitted from this text.

In tables I-IV following will be found the values assumed by *h, c* and the thresholds S for the lighter and heavier judgments for both subjects and for the original and the recalculated series. Tables I and II give respectively the values for the lighter and heavier judgments for subject I; tables III and IV give the same values for subject II. The tables

TABLE I

| Series | $h_1$ | | $c_1$ | | $S_1$ | |
|---|---|---|---|---|---|---|
| | Original | Recalculated | Original | Recalculated | Original | Recalculated |
| I | 0.11051 | 0.10881 | 10.337 | 10.172 | 94.54 | 93.90 |
| II | 0.10769 | 0.11007 | 10.108 | 10.323 | 93.86 | 93.78 |
| III | 0.11267 | 0.13983 | 10.617 | 13.134 | 94.23 | 93.92 |
| IV | 0.10976 | 0.13866 | 10.442 | 13.211 | 95.13 | 95.27 |
| V | 0.11714 | 0.12113 | 10.988 | 11.330 | 93.81 | 93.54 |
| VI | 0.10432 | 0.09497 | 9.814 | 8.911 | 94.08 | 93.84 |
| VII | 0.11156 | 0.10983 | 10.586 | 10.435 | 94.89 | 95.01 |
| VIII | 0.11379 | 0.11728 | 10.796 | 11.096 | 94.88 | 94.62 |
| IX | 0.11477 | 0.11727 | 10.889 | 11.136 | 94.88 | 94.96 |
| X | 0.12274 | 0.11852 | 11.790 | 11.382 | 96.06 | 96.04 |
| XI | 0.09902 | 0.09253 | 9.351 | 8.718 | 94.43 | 94.22 |
| XII | 0.12287 | 0.12361 | 11.670 | 11.742 | 94.99 | 95.00 |
| XIII | 0.12794 | 0.12794 | 12.251 | 12.251 | 95.76 | 95.76 |
| XIV | 0.12446 | 0.13276 | 11.795 | 12.562 | 94.77 | 94.62 |

all show a similar arrangement. In the first columns are found the numbers of the groups of 100 judgments in the order in which they were taken. In the next two columns are found the values of *h;* the first of these being of the original series with the six comparison weights and the next shows similar values for the recalculated series in which only the four central intensities of the comparison stimulus were considered. In the next two columns are found, in a like manner, the original and recalculated values of *c;* and the last

two columns show a similar arrangement of the values assumed by $S_1$ and by $S_2$ or the threshold in the direction of decrease and increase respectively.

TABLE II

| Series | $h_2$ | | $c_2$ | | $S_2$ | |
|---|---|---|---|---|---|---|
| | Original | Recalculated | Original | Recalculated | Original | Recalculated |
| I | 0.11371 | 0.11264 | 11.374 | 11.295 | 100.03 | 100.28 |
| II | 0.11914 | 0.12153 | 11.888 | 12.114 | 99.78 | 99.68 |
| III | 0.11629 | 0.11917 | 11.537 | 11.801 | 99.21 | 99.03 |
| IV | 0.11734 | 0.13062 | 11.731 | 13.043 | 99.98 | 99.86 |
| V | 0.13496 | 0.14688 | 13.406 | 14.555 | 99.33 | 99.10 |
| VI | 0.11059 | 0.10519 | 11.004 | 10.510 | 99.50 | 99.92 |
| VII | 0.11393 | 0.13854 | 11.362 | 13.772 | 99.72 | 99.41 |
| VIII | 0.11641 | 0.11936 | 11.560 | 11.832 | 99.30 | 99.14 |
| IX | 0.13843 | 0.14136 | 13.724 | 14.054 | 99.14 | 99.42 |
| X | 0.13364 | 0.18659 | 13.351 | 18.520 | 99.90 | 99.26 |
| XI | 0.12086 | 0.12239 | 12.081 | 12.257 | 99.95 | 100.14 |
| XII | 0.12004 | 0.12401 | 12.012 | 12.388 | 100.07 | 99.89 |
| XIII | 0.14643 | 0.13761 | 14.725 | 13.916 | 100.56 | 100.91 |
| XIV | 0.12390 | 0.15294 | 12.421 | 15.202 | 100.25 | 99.40 |

TABLE III

| Series | $h_1$ | | $c_1$ | | $S_1$ | |
|---|---|---|---|---|---|---|
| | Original | Recalculated | Original | Recalculated | Original | Recalculated |
| I | 0.07570 | 0.06768 | 7.023 | 6.266 | 92.77 | 92.59 |
| II | 0.09224 | 0.08544 | 8.728 | 8.074 | 94.62 | 94.50 |
| III | 0.10859 | 0.12446 | 10.338 | 11.816 | 95.22 | 94.94 |
| IV | 0.13786 | 0.14169 | 13.018 | 13.356 | 94.42 | 94.26 |
| V | 0.10672 | 0.08403 | 10.087 | 7.981 | 94.52 | 94.98 |
| VI | 0.13459 | 0.13459 | 12.587 | 12.587 | 93.52 | 93.52 |
| VII | 0.10743 | 0.11189 | 10.070 | 10.455 | 93.74 | 93.44 |
| VIII | 0.12512 | 0.12343 | 11.604 | 11.428 | 92.73 | 92.59 |
| IX | 0.12402 | 0.12918 | 11.511 | 11.981 | 92.81 | 92.75 |
| X | 0.11502 | 0.11286 | 10.797 | 10.563 | 93.87 | 93.60 |
| XI | 0.10223 | 0.10223 | 9.460 | 9.460 | 92.53 | 92.53 |
| XII | 0.11750 | 0.12038 | 11.126 | 11.384 | 94.70 | 94.58 |
| XIII | 0.13071 | 0.13337 | 12.307 | 12.569 | 94.16 | 94.24 |
| XIV | 0.12920 | 0.12841 | 12.239 | 12.158 | 94.73 | 94.68 |

TABLE IV

| Series | $h_2$ | | $c_2$ | | $S_2$ | |
|---|---|---|---|---|---|---|
| | Original | Recalculated | Original | Recalculated | Original | Recalculated |
| I | 0.08126 | 0.09794 | 7.936 | 9.538 | 97.67 | 97.38 |
| II | 0.09763 | 0.10472 | 9.706 | 10.402 | 99.42 | 99.33 |
| III | 0.10666 | 0.11693 | 10.500 | 11.483 | 98.45 | 98.21 |
| IV | 0.12340 | 0.14405 | 12.108 | 14.106 | 98.12 | 97.92 |
| V | 0.12580 | 0.12949 | 12.291 | 12.635 | 97.70 | 97.57 |
| VI | 0.14124 | 0.16361 | 13.812 | 15.913 | 97.80 | 97.26 |
| VII | 0.11233 | 0.11980 | 11.090 | 11.788 | 98.72 | 98.40 |
| VIII | 0.11112 | 0.12129 | 10.937 | 11.880 | 98.43 | 97.94 |
| IX | 0.12023 | 0.12279 | 11.816 | 12.062 | 98.28 | 98.24 |
| X | 0.10467 | 0.10708 | 10.328 | 10.556 | 98.67 | 98.58 |
| XI | 0.12377 | 0.11961 | 12.161 | 11.771 | 98.25 | 98.41 |
| XII | 0.11801 | 0.11673 | 11.865 | 11.737 | 100.54 | 100.55 |
| XIII | 0.10686 | 0.09516 | 10.722 | 9.634 | 100.34 | 101.24 |
| XIV | 0.13650 | 0.14695 | 13.648 | 14.649 | 99.98 | 99.69 |

An examination of these tables shows that, although the values of the *h* and S in the two calculations are by no means identical, still, on the whole, they show a rather close resemblance to one another. The course of the values of the *h* for both subjects show a very similar curve for the recalculated series as they do in the original. Hence we would have been able to point out the effects of progressive practice quite as readily if we had considered only the four central values.[10]

But a psychophysical method is primarily a prescription for collecting data and for evaluating them in such a way that the result enables us to compare the sensitivity of different subjects, or of the same subject at different times, or under different conditions. Hence the thresholds are the important quantities in any psychophysical method; and therefore, they are the values which must come in for our greatest consideration. The difference between the averages of the thresholds of the two series is found in no case to exceed one-fifth of a gram. The averages are:

[10] Fernberger, *ibid,* pp. 41-46.

| | | $S_1$ | $S_2$ |
|---|---|---|---|
| Subject I | Original | 94.736 | 99.766 |
| | Recalculated | 94.606 | 99.674 |
| | Difference | 0.130 | 0.092 |
| Subject II | Original | 93.880 | 98.741 |
| | Recalculated | 93.800 | 98.623 |
| | Difference | 0.080 | 0.118 |

These differences are of such little significance that they may practically be disregarded. When we examine the measure of sensitivity $\left(\frac{S_2 - S_1}{2}\right)$ we find a very surprising uniformity indeed. These values are:

| | | |
|---|---|---|
| Subject I | Original | 2.51 |
| | Recalculated | 2.53 |
| Subject II | Original | 2.43 |
| | Recalculated | 2.41 |

Here we find an agreement which is much closer than might have been expected considering that when we eliminate the data of the 108 and 84 gram weights, we are disregarding one-third of the material that is the basis of this experimental study.

When we consider the individual values that the threshold assumes in the groups of 100 reactions, we find, however, a very different state of affairs and one which does not by any means show the uniformity pointed out above. These values vary from identity (Subject I, $S_1$, XIII) to a difference of 0.90 grams (Subject II, $S_2$, XIII). These differences are of course due to variations of a chance character in the observed relative frequencies on one or both of the stimuli which we

have disregarded. From the averages of these same values, however, it would seem that in an extended study these variations tend to cancel one another so that the aggregate thresholds and the measure of sensitivity show extremely little variation.

When we consider the form of the curves of the psychometric functions for the heavier and lighter judgments, it is evident that the central part of the curve is the most important.[11] There is a very sharp rise at the central values of our comparison stimuli; while at the ends of the curve, the rise is very gradual—towards unity in one direction and approaching the abscissa asymptotically in the other. Hence these extreme values will not exert as much influence on the value of the constants, which determine the form and position of these curves, as will the central values where the rise is very rapid; and so they may be more readily disregarded.

There is another very practical experimental argument in favor of the elimination of the extreme intensities of the comparison stimuli. It has been pointed out that in any experimental series, no matter how carefully planned, there are certain stimuli which serve as landmarks and which lead to a more or less unconscious learning of the order of presentation of the stimuli.[12] The basis of the method of constant stimuli is that the subject has no knowledge of the objective relation of the stimuli presented to him. To obviate this learning of the experimental order, the procedure has been to change it from time to time when the subject reports or gives evidences of having acquired a knowledge of such land-marks. In the study referred to above, such land-marks were always found to be either the 84 or 108 gram weights—the extreme stimuli. Hence the elimination of these would make it much more difficult for the subject to acquire any knowledge of the objective relation of the stimuli and so, in this case, we should have the method in its purest form. A study of the tables will further support this contention. It will be noticed that, on the whole, the values of $h$ in the recalculated series, where the extreme stimuli were omitted, are greater than in the original series. This matter was referred to Professor F. M. Urban of the University of Pennsylvania and the following is a quotation from a personal letter from him. "On general grounds one ought to expect that omitting the results for 84

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[11] F. M. Urban, The method of constant stimuli and its generalizations. *Psychol. Rev.*, XVII, 1910. 229-259.
Fernberger, *ibid*, 29f.

[12] Fernberger, *ibid*, 12.

and 108 grams might just as well diminish as increase the final result. As a matter of fact this is not so. The explanation may be as follows. The addition of the extreme comparison weights has a tendency to reduce the values of *h*. This they could not do unless the probabilities on these extreme weights were smaller (or larger) than one would expect from the results in the middle part of the series of stimuli. In other words, the extreme comparison weights are not judged in the same way as the others. This is a further argument in favor of omitting the extreme values, since they tend to make the conditions less uniform."

Hence it is evident that there are very sound theoretical and practical arguments in favor of the elimination of the extreme stimuli in the method of constant stimuli in the form under consideration. The calculations show that such a shortening of the series makes, in an extended experiment, practically no difference in the measure of sensitivity of the subject, although the point of subjective equality is somewhat, although not considerably, changed. In short series there prove to be relatively great and by no means regular variations. But it is doubtful if the method of constant stimuli itself may be successfully applied in a short series, inasmuch as the influence of progressive practice changes the final values to a considerable extent. Hence we may consider this method as one which should properly be applied only as an extended experimental procedure; and under these conditions it would seem that the extreme values of the comparison stimuli may be eliminated without materially affecting the final result. Such an elimination not only shortens the calculations to some extent but,—what is obviously of much greater importance,—it reduces the time and labor that must be expended in the acquiring of the data upon which the calculations are based by practically one-third, without, formally at least, affecting the accuracy of the determinations of the sensitivity of the subject.